

JEE-MAIN EXAMINATION – JANUARY 2026

(HELD ON WEDNESDAY 21st JANUARY 2026)

TIME : 9:00 AM TO 12:00 NOON

MATHEMATICS

SECTION-A

1. If the domain of the function

$$f(x) = \cos^{-1}\left(\frac{2x-5}{11-3x}\right) + \sin^{-1}(2x^2 - 3x + 1) \text{ is the}$$

interval $[\alpha, \beta]$, then $\alpha + 2\beta$ is equal to :

- (1) 1 (2) 3
(3) 5 (4) 2

Ans. (2)

Sol. $f(x) = \cos^{-1}\left(\frac{2x-5}{11-3x}\right) + \sin^{-1}(2x^2 - 3x + 1)$

$$-1 \leq \frac{2x-5}{11-3x} \leq 1$$

$$-1 \leq 2x^2 - 3x + 1 \leq 1$$

$$2x^2 - 3x + 2 \geq 0, 2x^2 - 3x \leq 0$$

$$x \in \left[0, \frac{3}{2}\right] \dots (i)$$

$$\frac{2x-5}{11-3x} + 1 \geq 0$$

$$\frac{2x-5+11-3x}{11-3x} \geq 0$$

$$\frac{6-x}{11-3x} \geq 0$$

$$+ \quad - \quad +$$

$$x \in \left(-\infty, \frac{16}{5}\right] \cup \left(\frac{11}{3}, \infty\right)$$

$$x \in \left(-\infty, \frac{11}{3}\right) \cup [6, \infty)$$

intersection

$$x \in \left(-\infty, \frac{16}{5}\right] \cup [6, \infty) \dots (ii)$$

$$\text{Intersection of (i) \& (ii) } x \in \left[0, \frac{3}{2}\right]$$

$$\alpha = 0, \beta = \frac{3}{2} \Rightarrow \alpha + 2\beta = 3$$

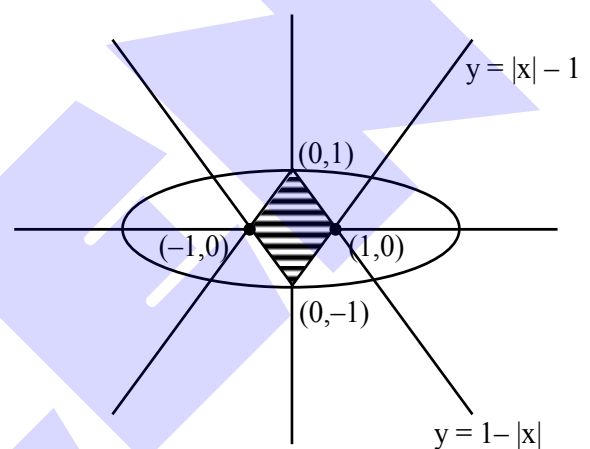
TEST PAPER WITH SOLUTION

2. The area of the region, inside the ellipse $x^2 + 4y^2 = 4$ and outside the region bounded by the curves $y = |x| - 1$ and $y = 1 - |x|$, is :

- (1) $2(\pi - 1)$ (2) $2\pi - \frac{1}{2}$
(3) $3(\pi - 1)$ (4) $2\pi - 1$

Ans. (1)

Sol.



Required area = area of ellipse – shaded area

$$= \pi \times 2 \times 1 - 4 \left(\frac{1}{2} \times 1 \times 1\right) = 2\pi - 2$$

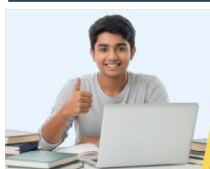
3. The number of relations, defined on the set $\{a, b, c, d\}$, which are both reflexive and symmetric, is equal to:

- (1) 256 (2) 16
(3) 1024 (4) 64

Ans. (4)

Sol. Number of relation which are reflex and sym. both
 $= 1^4 \times 2^6 = 64$

- | | | | |
|--------|--------|--------|--------|
| (a, a) | (a, b) | (a, c) | (a, d) |
| (b, a) | (b, b) | (b, c) | (b, d) |
| (c, a) | (c, b) | (c, c) | (c, d) |
| (d, a) | (d, b) | (d, c) | (d, d) |



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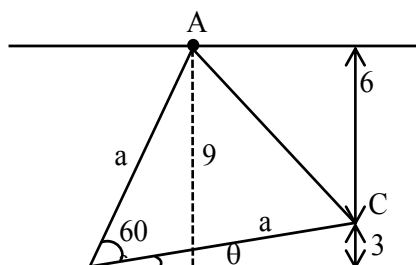
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4. Let a point A lie between the parallel lines L_1 and L_2 such that its distances from L_1 and L_2 are 6 and 3 units, respectively. Then the area (in sq. units) of the equilateral triangle ABC, where the points B and C lie on the lines L_1 and L_2 respectively, is :

- (1) $15\sqrt{6}$ (2) 27
(3) $21\sqrt{3}$ (4) $12\sqrt{2}$

Ans. (3)



Sol.

$$\sin \theta = \frac{3}{a}$$

$$\sin (60^\circ + \theta) = \frac{9}{a}$$

$$\frac{\sqrt{3}}{2} \cos \theta + \frac{1}{2} \sin \theta = \frac{9}{a}$$

$$\sqrt{3} \sqrt{1 - \frac{9}{a^2}} + \frac{3}{a} = \frac{18}{a}$$

$$a = \sqrt{84}$$

$$\text{Area of } \triangle ABC = \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} \times 84 = 21\sqrt{3}$$

5. Let $\vec{a} = -\hat{i} + 2\hat{j} + 2\hat{k}$, $\vec{b} = 8\hat{i} + 7\hat{j} - 3\hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} = \vec{b}$. If $\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = 4$, then $|\vec{a} + \vec{c}|^2$ is equal to :

- (1) 33 (2) 30
(3) 35 (4) 27

Ans. (4)

Sol. $\vec{a} = -\hat{i} + 2\hat{j} + 2\hat{k}$

$$\vec{b} = 8\hat{i} + 7\hat{j} - 3\hat{k}$$

$$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

$$\vec{a} \times \vec{c} = \vec{b} \Rightarrow (2c_3 - 2c_2)\hat{i} + (c_3 + 2c_1)\hat{j} - (c_2 + 2c_1)\hat{k} = 8\hat{i} + 7\hat{j} - 3\hat{k}$$

$$2c_3 - 2c_2 = 8, c_3 + 2c_1 = 7, c_2 + 2c_1 = 3$$

$$(c_1\hat{i} + c_2\hat{j} + c_3\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = 4$$

$$\Rightarrow c_1 + c_2 + c_3 = 4, c_1 = 2, c_2 = -1, c_3 = 3$$

$$|\vec{a} + \vec{c}|^2 = |\hat{i} + \hat{j} + 5\hat{k}|^2 = 27$$

6. Let $a_1, a_2, a_3, \dots$ be a G.P. of increasing positive terms such that $a_2 a_3 a_4 = 64$ and $a_1 + a_3 + a_5 = \frac{813}{7}$.

Then $a_3 + a_5 + a_7$ is equal to :

- (1) 3256 (2) 3252
(3) 3244 (4) 3248

Ans. (2)

Sol. $ar \cdot ar^2 \cdot ar^3 = 64$

$$a^3 r^6 = 64 \Rightarrow ar^2 = 4$$

$$a + ar^2 + ar^4 = \frac{813}{7}$$

$$r^2 = 28$$

$$ar^2 + ar^4 + ar^6 = ?$$

$$ar^2 (1 + r^2 + r^4) = 4 (1 + 28 + 784) = 3252$$

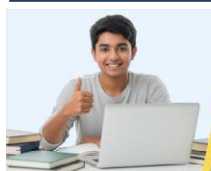
7. Let $\vec{c}$ and $\vec{d}$ be vectors such that $|\vec{c} + \vec{d}| = \sqrt{29}$ and $\vec{c} \times (2\hat{i} + 3\hat{j} + 4\hat{k}) = (2\hat{i} + 3\hat{j} + 4\hat{k}) \times \vec{d}$. If $\lambda_1, \lambda_2 (\lambda_1 > \lambda_2)$ are the possible values of $(\vec{c} + \vec{d}) \cdot (-7\hat{i} + 2\hat{j} + 3\hat{k})$, then the equation

$$K^2 x^2 + (K^2 - 5K + \lambda_1)xy + \left(3K + \frac{\lambda_2}{2}\right)y^2 - 8x + 12y + \lambda_2 = 0$$

represents a circle, for k equal to :

- (1) 4 (2) 1
(3) -1 (4) 2

Ans. (2)



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Sol. $|\vec{c} + \vec{d}| = \sqrt{29}$

$$\vec{c} + \vec{d} = \lambda ((2\hat{i} + 3\hat{j} + 4\hat{k}))$$

$$\lambda = \pm 1$$

$$\lambda(-14 + 6 + 12) = 4\lambda, \lambda_1 = 4, \lambda_2 = -4$$

$$k^2x^2 + (k^2 - 5k + 4)xy + (3k - 2)y^2 - 8x + 12y - 4 = 0$$

is circle

$$k^2 - 5k + 4 = 0 \Rightarrow k = 1, 4$$

$$k^2 = 3k - 2 \Rightarrow k = 1, 2$$

$$k = 1$$

8. Let $y = y(x)$ be the solution curve of the differential equation $(1 + x^2)dy + (y - \tan^{-1}x)dx = 0$, $y(0) = 1$. Then the value of $y(1)$ is :

- (1) $\frac{2}{e^4} + \frac{\pi}{4} - 1$ (2) $\frac{2}{e^4} - \frac{\pi}{4} - 1$
 (3) $\frac{4}{e^4} + \frac{\pi}{2} - 1$ (4) $\frac{4}{e^4} - \frac{\pi}{2} - 1$

Ans. (1)

Sol. $\frac{dy}{dx} + \frac{y}{x^2 + 1} = \frac{\tan^{-1}x}{x^2 + 1}$

$$\text{I.f.} = e^{\tan^{-1}x}$$

$$y \times e^{\tan^{-1}x} = \int e^{\tan^{-1}x} \cdot \frac{\tan^{-1}x}{1 + x^2} dx$$

$$y \times e^{\tan^{-1}x} = \tan^{-1}x (e^{\tan^{-1}x}) - e^{\tan^{-1}x} + c$$

$$y(0) = 1 \Rightarrow c = 2$$

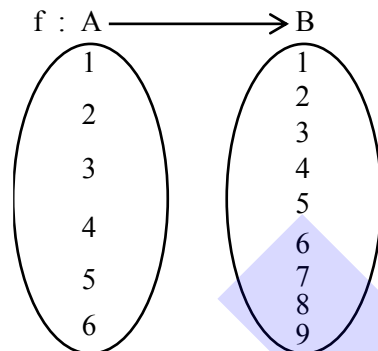
$$y(1) = \frac{2}{e^{\pi/4}} + \frac{\pi}{4} - 1$$

9. The number of strictly increasing functions f from the set $\{1, 2, 3, 4, 5, 6\}$ to the set $\{1, 2, 3, \dots, 9\}$ such that $f(i) \neq i$ for $1 \leq i \leq 6$, is equal to :

- (1) 21 (2) 27
 (3) 22 (4) 28

Ans. (4)

- Sol.** $f(i) \neq i$, $f(x)$ is strictly increasing function $f: A \rightarrow B$, where $A = \{1, 2, 3, \dots, 6\}$
 $B = \{1, 2, 3, \dots, 9\}$, then number of function $f: A \rightarrow B$ is equal to



$$f(i) \neq i \text{ Case - i } f(1) = 2 \Rightarrow {}^7C_5 = 21$$

$$\text{Case- ii } f(1) = 3 \Rightarrow {}^6C_5 = 6$$

$$\text{Case- iii } f(1) = 4 \Rightarrow {}^5C_5 = 1$$

$$\text{No of function A to B} = 21 + 6 + 1 = 28$$

10. Let $f: \mathbb{R} \rightarrow (0, \infty)$ be a twice differentiable function such that $f(3) = 18$, $f'(3) = 0$ and $f''(3) = 4$.

$$\text{Then } \lim_{x \rightarrow 1} \left(\log_e \left(\frac{f(2+x)}{f(3)} \right)^{\frac{18}{(x-1)^2}} \right) \text{ is equal to :}$$

- (1) 1 (2) 9
 (3) 2 (4) 18

Ans. (3)

Sol. Let $T = \lim_{x \rightarrow 1} \left(\frac{f(x+2)}{f(3)} \right)^{\frac{18}{(x-1)^2}}$; 1^∞ form

$$\Rightarrow T = e^{\lim_{x \rightarrow 1} \frac{18}{(x-1)^2} \left(\frac{f(x+2) - f(3)}{f(3)} \right)}$$

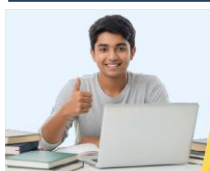
$$\Rightarrow T = e^{\lim_{x \rightarrow 1} \frac{18}{(x-1)^2} \left(\frac{f(x+2) - f(3)}{18} \right)}$$

$$\Rightarrow T = e^{\lim_{x \rightarrow 1} \left(\frac{f(x+2) - f(3)}{(x-1)^2} \right) \frac{0}{0} \text{ form}} \text{ apply L' pital}$$

$$\Rightarrow T = e^{\lim_{x \rightarrow 1} \frac{f'(x+2)}{2(x-1)} \frac{0}{0} \text{ form}} \text{ apply L' pital}$$

$$\Rightarrow T = e^{\lim_{x \rightarrow 1} \frac{f''(x+2)}{2} \frac{4}{2}} = e^2 = e^2$$

$$\Rightarrow \log_e(T) = 2$$



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11. Let the foci of hyperbola coincide with the foci of the ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$. If the eccentricity of the hyperbola is 5, then the length of its latus rectum is:

- (1) 12 (2) 16
(3) $\frac{96}{\sqrt{5}}$ (4) $24\sqrt{5}$

Ans. (3)

Sol. Let e_1 be eccentricity of ellipse

$$\Rightarrow e_1 = \sqrt{1 - \frac{16}{36}} = \sqrt{1 - \frac{4}{9}} = \frac{\sqrt{5}}{3}$$

$$\text{So } ae_1 = 6 \cdot \frac{\sqrt{5}}{3} = 2\sqrt{5}$$

$$\text{Now H: } \frac{x^2}{p^2} - \frac{y^2}{q^2} = 1$$

$$p \cdot e = ae_1$$

$$p \cdot 5 = 2\sqrt{5}$$

$$p = \frac{2}{\sqrt{5}} \Rightarrow e^2 = 1 + \frac{q^2}{p^2} \Rightarrow 25 = 1 + \frac{5q^2}{4} \Rightarrow q^2 = \frac{96}{5}$$

$$\text{So length of LR } \frac{2q^2}{p} = \frac{96}{\sqrt{5}}$$

12. The value of $\int_{-\pi/6}^{\pi/6} \left(\frac{\pi + 4x^{11}}{1 - \sin(|x| + \pi/6)} \right) dx$ is equal to

- (1) 2π (2) 4π
(3) 8π (4) 6π

Ans. (2)

$$\text{Sol. } = 2\pi \int_0^{\pi/6} \frac{1}{1 - \sin\left(x + \frac{\pi}{6}\right)} dx \text{ let } x + \frac{\pi}{6} = t \Rightarrow dx = dt$$

$$= 2\pi \int_{\pi/6}^{\pi/3} \frac{dt}{1 - \sin t} = 2\pi \int_{\pi/6}^{\pi/3} \frac{1 + \sin t}{\cos^2 t} dt$$

$$= 2\pi \left[\int_{\pi/6}^{\pi/3} \sec^2 t dt + \int_{\pi/6}^{\pi/3} \sec t \tan t dt \right]$$

$$= 2\pi \left[(\tan t)_{\pi/6}^{\pi/3} + (\sec t)_{\pi/6}^{\pi/3} \right]$$

$$= 2\pi \left[\left(\sqrt{3} - \frac{1}{\sqrt{3}} \right) + \left(2 - \frac{2}{\sqrt{3}} \right) \right]$$

$$= 2\pi \left[\sqrt{3} + 2 - \sqrt{3} \right] = 4\pi$$

13. Let the mean and variance of 7 observations 2, 4, 10, x, 12, 14, y, $x > y$, be 8 and 16 respectively. Two numbers are chosen from $\{1, 2, 3, x-4, y, 5\}$ one after another without replacement, then the probability, that the smaller number among the two chosen numbers is less than 4, is:

- (1) $\frac{3}{5}$ (2) $\frac{4}{5}$
(3) $\frac{2}{5}$ (4) $\frac{1}{3}$

Ans. (2)

Sol. Mean $(\bar{x}) = 8$ (Given)

$$\Rightarrow \frac{2 + 4 + 10 + x + 12 + 14 + y}{7} = 8$$

$$\Rightarrow x + y = 14 \dots (1) \dots$$

Variance $(\sigma^2) = 16$ (Given)

$$\Rightarrow 16 = \frac{2^2 + 4^2 + 10^2 + x^2 + 12^2 + 14^2 + y^2}{7} - 8^2$$

$$\Rightarrow x^2 + y^2 = 100 \dots (2) \dots$$

$$\therefore (x + y)^2 = x^2 + y^2 + 2xy$$

$$\Rightarrow xy = 48 \text{ (sum is 14 product is 48)}$$

Since problem states $x > y$

$$\therefore x = 8 \text{ and } y = 6$$

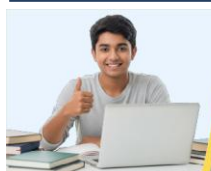
Now set $X = \{1, 2, 3, 4, 6, 5\}$

Now we choose two numbers one after another without replacement total outcomes = $6 \times 5 = 30$

We want the prob. That the smaller number among the two is less than 4

$$P(\text{smaller} < 4) = 1 - P(\text{smaller} \geq 4)$$

$$= 1 - \frac{6}{30} = \frac{4}{5}$$



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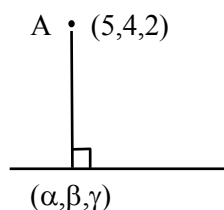
14. Let (α, β, γ) be the co-ordinates of the foot of the perpendicular drawn from the point $(5, 4, 2)$ on the line $\vec{r} = (-\hat{i} + 3\hat{j} + \hat{k}) + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$.

Then the length of the projection of the vector $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ on the vector $6\hat{i} + 2\hat{j} + 3\hat{k}$ is :

- (1) $\frac{15}{7}$ (2) 4
(3) $\frac{18}{7}$ (4) 3

Ans. (3)

Sol.



$$\vec{r} = (-\hat{i} + 3\hat{j} + \hat{k}) + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$$

$$\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1} = \lambda$$

Any general point P on the line is

$$(2\lambda - 1, 3\lambda + 3, -\lambda + 1)$$

Let the given point is A $(5, 4, 2)$

$$\vec{AP} = (2\lambda - 6)\hat{i} + (3\lambda - 1)\hat{j} + (-\lambda - 1)\hat{k}$$

$$\therefore \vec{AP} \perp \text{Line (L)}$$

$$\therefore \vec{AP} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = 0$$

$$2(2\lambda - 6) + 3(3\lambda - 1) - 1(-\lambda - 1) = 0$$

$$\Rightarrow \lambda = 1$$

$$\therefore \alpha = 1$$

$$\beta = 6$$

$$\gamma = 0$$

$$\text{Let the vector } \vec{u} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$$

$$\vec{u} = \hat{i} + 6\hat{j} + 0\hat{k}$$

$$\& \vec{w} = 6\hat{i} + 2\hat{j} + 3\hat{k}$$

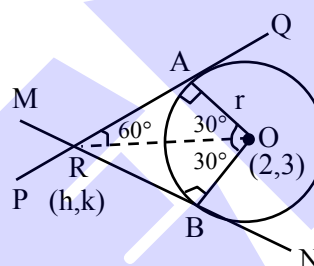
$$\text{So projection} = \frac{|\vec{u} \cdot \vec{w}|}{|\vec{w}|} = \frac{18}{7}$$

15. Let PQ and MN be two straight lines touching the circle $x^2 + y^2 - 4x - 6y - 3 = 0$ at the points A and B respectively. Let O be the centre of the circle and $\angle AOB = \pi/3$. Then the locus of the point of intersection of the lines PQ and MN is:

- (1) $3(x^2 + y^2) - 18x - 12y + 25 = 0$
(2) $x^2 + y^2 - 12x - 18y - 25 = 0$
(3) $x^2 + y^2 - 18x - 12y - 25 = 0$
(4) $3(x^2 + y^2) - 12x - 18y - 25 = 0$

Ans. (4)

Sol. Given circle



$$x^2 + y^2 - 4x - 6y - 3 = 0$$

$$C(2, 3) \& r = 4$$

$$\cos 30^\circ = \frac{r}{OR} = \frac{4}{OR}$$

$$\Rightarrow OR = \frac{8}{\sqrt{3}}$$

Now

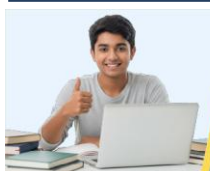
$$OR^2 = (h - 2)^2 + (k - 3)^2$$

$$\Rightarrow 3(x^2 + y^2) - 12x - 18y - 25 = 0$$

16. If the coefficient of x in the expansion of $(ax^2 + bx + c)(1 - 2x)^{26}$ is -56 and the coefficients of x^2 and x^3 are both zero, then $a + b + c$ is equal to

- (1) 1300 (2) 1500
(3) 1403 (4) 1483

Ans. (3)



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Sol. $(ax^2 + bc + c) \sum_{r=0}^{26} {}^{26}C_r (-2x)^r$

Coeff. of x^2 : $a \cdot {}^{26}C_0 (-2)^0 + b \cdot {}^{26}C_1 (-2) + c \cdot {}^{26}C_2 (-2)^2 = 0$

$\Rightarrow a - 52b + 1300c = 0 \dots (1)$

Coeff. of x^3 : $a \cdot {}^{26}C_1 (-2) + b \cdot {}^{26}C_2 (-2)^2 + c \cdot {}^{26}C_3 (-2)^3 = 0$

$\Rightarrow -52a + 1300b - 20800c = 0 \dots (2)$

Coeff. of $x = -56$

$\Rightarrow b \cdot {}^{26}C_0 (-2)^0 + c \cdot {}^{26}C_1 (-2)^1 = -56$

$b - 52c = -56 \dots (3)$

After solving (1), (2) & (3)

$a = 1300, b = 100, c = 3$

$\Rightarrow a + b + c = 1403$

17. If $x^2 + x + 1 = 0$, then the value of $\left(x + \frac{1}{x}\right)^4 + \left(x^2 + \frac{1}{x^2}\right)^4 + \left(x^3 + \frac{1}{x^3}\right)^4 + \dots + \left(x^{25} + \frac{1}{x^{25}}\right)^4$ is :

(1) 128

(2) 162

(3) 175

(4) 145

Ans. (4)

Sol. $x^2 + x + 1 = 0$

$\Rightarrow x = \omega \text{ or } \omega^2$

$\therefore \alpha = \omega \text{ & } \beta = \omega^2$

$= (\omega + \omega^2)^4 + (\omega^2 + \omega^4)^4 + (\omega^3 + \omega^6)^4 + \dots + (\omega^{25} + \omega^{50})^4$

$= [(\omega + \omega^2)^4 + (\omega^2 + \omega^4)^4 + (\omega^4 + \omega^8)^4 + \dots + (\omega^{25} + \omega^{50})^4] + [(\omega^3 + \omega^6)^4 + (\omega^6 + \omega^{12})^4 + (\omega^9 + \omega^{18})^4 + \dots + (\omega^{24} + \omega^{48})^4]$

$= \underbrace{[1 + 1 + 1 + \dots + 1]}_{17 \text{ times}} + \underbrace{[(1+1)^4 + (1+1)^4 + \dots + (1+1)^4]}_{8 \text{ times}}$

$= 17 + 128$

$= 145$

18. The value of $\operatorname{cosec} 10^\circ - \sqrt{3} \sec 10^\circ$ is equal to:

(1) 4

(2) 2

(3) 8

(4) 6

Ans. (1)

Sol. $= \frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ}$

$= \frac{\cos 10^\circ - \sqrt{3} \sin 10^\circ}{\sin 10^\circ \cos 10^\circ}$

$= 4 \left[\frac{\frac{1}{2} \cos 10^\circ - \frac{\sqrt{3}}{2} \sin 10^\circ}{2 \sin 10^\circ \cos 10^\circ} \right]$

$= 4 \left[\frac{\sin(30^\circ - 10^\circ)}{\sin 20^\circ} \right]$

$= 4$

19. The sum of all the roots of the equation $(x-1)^2 - 5|x-1| + 6 = 0$, is:

(1) 4

(2) 3

(3) 1

(4) 5

Ans. (1)

Sol. Let $|x-1| = t$

$t^2 - 5t + 6 = 0$

$t = 2 \text{ & } t = 3$

$|x-1| = 2 \text{ & } |x-1| = 3$

$x-1 \pm 2 \text{ & } x-1 = \pm 3$

$x = 1 \pm 2 \text{ & } x = 1 \pm 3$

$\therefore \text{root} = 3, -1, 4, -2$

$\therefore \text{Sum of root} = 3 + (-1) + 4 + (-2) = 4.$

20. Let O be the vertex of the parabola $x^2 = 4y$ and Q be any point on it. Let the locus of the point P, which divides the line segment OQ internally in the ratio 2:3 be the conic C. Then the equation of the chord of C, which is bisected at the point (1, 2), is:

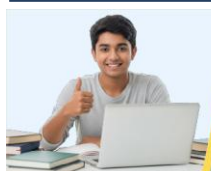
(1) $5x - y - 3 = 0$

(2) $4x - 5y + 6 = 0$

(4) $x - 2y + 3 = 0$

(4) $5x - 4y + 3 = 0$

Ans. (4)

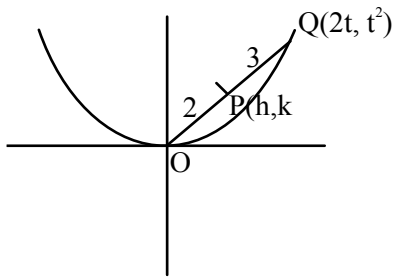


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Sol. $h = \frac{4t}{5}$



$$k = \frac{2t^2}{5} = \frac{2}{5} \left(\frac{5h}{4} \right)^2$$

$$8k = 5h^2$$

$$\Rightarrow 5x^2 = 8y$$

$$T = S_1$$

$$5(xx_1) - 4(y + y_1) = 5x_1^2 - 8y_1$$

$$5(x) - 4(y + 2) = 5 - 8.2$$

$$5x - 4y + 3 = 0$$

SECTION-B

21. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a twice differentiable function such that the quadratic equation $f(x)m^2 - 2f'(x)m + f''(x) = 0$ in m , has two equal roots for every $x \in \mathbb{R}$. If $f(0) = 1$, $f'(0) = 2$ and (α, β) is the largest interval in which the function $f(\log_e x - x)$ is increasing, then $\alpha + \beta$ is equal to

Ans. (1)

Sol. Given quadratic equation has equal roots, thus

$$D = 0 \Rightarrow (f'(x))^2 = f''(x) \cdot f(x)$$

$$\frac{f'(x)}{f(x)} = \frac{f''(x)}{f'(x)}$$

Integrate

$$\ln(f(x)) = \ln(f'(x)) + \ln C \Rightarrow f(x) = c \cdot f'(x)$$

Put $x = 0$

$$1 = c \cdot 2 \Rightarrow c = \frac{1}{2}$$

$$\text{Now } 2f(x) = f'(x)$$

$$\Rightarrow \frac{f'(x)}{f(x)} = 2$$

Integrate

$$\ln(f(x)) = 2x + d$$

$$\Rightarrow d = 0$$

$$\Rightarrow \ln(f(x)) = 2x \Rightarrow f(x) = e^{2x}$$

$$\text{Now let } g(x) = f(\ln x - x) = e^{2(\ln x - x)}$$

$$\therefore g'(x) = 2e^{2(\ln x - x)} \left(\frac{1}{x} - 1 \right) \geq 3$$

$$\Rightarrow \frac{1-x}{x} \geq 0$$

$$\Rightarrow x \in (0, 1]$$

$$\Rightarrow \alpha = 0, \beta = 1$$

$$\alpha + \beta = 1.$$

22. Let $a_1 = 1$ and for $n \geq 1$, $a_{n+1} = \frac{1}{2}a_n + \frac{n^2 - 2n - 1}{n^2(n+1)^2}$. Then $\left| \sum_{n=1}^{\infty} \left(a_n - \frac{2}{n^2} \right) \right|$ is equal to _____.

Ans. (2)

Sol. $a_{n+1} - \frac{1}{2}a_n = \frac{n^2 - 2n - 1}{n^2(n+1)^2} = \frac{2n^2 - (n+1)^3}{n^2(n+1)^2}$

$$\Rightarrow a_{n+1} - \frac{1}{2}a_n = \frac{2}{(n+1)^2} - \frac{1}{n^2}$$

$$n=1 \quad a_2 - \frac{1}{2}a_1 = \frac{2}{2^2} - \frac{1}{1^2}$$

$$2 \left[a_3 - \frac{1}{2}a_2 = \frac{2}{3^2} - \frac{1}{2^2} \right]$$

$$2^2 \left[a_4 - \frac{1}{2}a_3 = \frac{2}{4^2} - \frac{1}{3^2} \right]$$

$\vdots$

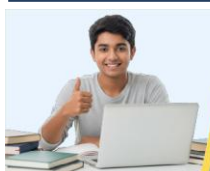
$$2^{n-2} \left[a_n - \frac{1}{2}a_{n-1} = \frac{2}{n^2} - \frac{1}{(n-1)^2} \right]$$

$$2^{n-1} \left[a_{n+1} - \frac{1}{2}a_n = \frac{2}{(n+1)^2} - \frac{1}{n^2} \right]$$

Adding

$$a_{n+1} = \frac{2}{(n+1)^2} - \frac{1}{2^n} \Rightarrow a_n = \frac{2}{n^2} - \frac{1}{2^{n-1}}$$

$$\Rightarrow \left| \sum_{n=1}^{\infty} \left(a_n - \frac{2}{n^2} \right) \right| \Rightarrow \left| \sum_{n=1}^{\infty} -\frac{1}{2^{n-1}} \right| = 2$$



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23. Let $S = \{(m, n) : m, n \in \{1, 2, 3, \dots, 50\}\}$. If the number of elements (m, n) in S such that $6^m + 9^n$ is a multiple of 5 is p and the number of elements (m, n) in S such that $m + n$ is a square of a prime number is q , then $p + q$ is equal to.....

Ans. (1333)

Sol. $S = \{1, 2, 3, \dots, 50\}$

$p = (6^m + 9^n)$ is divisible by 5

No. of ways

$$6^m = (5\lambda + 1)^m = 5k + 1$$

$$9^n = (10 - 1)^n = 10\mu - 1 \text{ if } n \text{ is odd}$$

$\Rightarrow n$ must be odd

$$10\mu + 1 \text{ if } n \text{ is even}$$

$$\Rightarrow \text{No. of ways} = 50 \times 25 = 1250$$

$q \Rightarrow (m + n)$ is square of a prime

	$m + n = 4$	$m + n = 9$	$m + n = 25$	$m + n = 49$
No. of	↓	↓	↓	↓
ways	3	8	24	48

$$q = 3 + 8 + 24 + 48 = 83$$

$$= p + q = 1250 + 83 = 1333$$

24. For some $\alpha, \beta \in \mathbb{R}$, let $A = \begin{bmatrix} \alpha & 2 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 1 & \beta \end{bmatrix}$ be such that $A^2 - 4A + 2I = B^2 - 3B + I = O$. Then $(\det(\text{adj}(A^3 - B^3)))^2$ is equal to

Ans. (225)

Sol. $\text{Tr}(A) = 4 \Rightarrow \alpha + 2 = 4 \Rightarrow \alpha = 2$

$$\text{Tr}(B) = 3 \Rightarrow \beta + 1 = 3 \Rightarrow \beta = 2$$

$$A^2 - 4A + 2I = O$$

$$A^3 = 4A^2 - 2A = 16A - 8I - 2A$$

$$14A - 8I$$

$$= \begin{bmatrix} 28 & 28 \\ 14 & 28 \end{bmatrix} + \begin{bmatrix} -8 & 0 \\ 0 & -8 \end{bmatrix}$$

$$= A^3 = \begin{bmatrix} 20 & 28 \\ 14 & 20 \end{bmatrix}$$

$$B^2 - 3B + I = O$$

$$B^2 = 3B - I$$

$$\Rightarrow B^3 = 3B^2 - B = 3(3B - I) - B = 8B - 3I$$

$$B^3 = \begin{pmatrix} 8 & 8 \\ 8 & 16 \end{pmatrix} + \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix} = \begin{pmatrix} 5 & 8 \\ 8 & 13 \end{pmatrix}$$

$$A^3 - B^3 = \begin{pmatrix} 20 & 28 \\ 14 & 20 \end{pmatrix} - \begin{pmatrix} 5 & 8 \\ 8 & 13 \end{pmatrix} = \begin{pmatrix} 15 & 20 \\ 6 & 7 \end{pmatrix}$$

$$\Rightarrow |A^3 - B^3| = 105 - 120 = -15$$

$$\Rightarrow |\text{adj}(A^3 - B^3)| = |A^3 - B^3| = -15$$

$$\Rightarrow |\text{adj}(A^3 - B^3)|^2 = 225$$

25. $6 \int_0^\pi (\sin 3x + \sin 2x + \sin x) dx$ is equal to.....

Ans. (17)

Sol. $6 \int_0^\pi |2 \sin 2x \cos x + \sin 2x| dx$

$$6 \int_0^\pi |4 \sin x \cos^2 x + 2 \sin x \cos x| dx$$

$$I = 12 \int_0^\pi \sin x |2 \cos^2 x + \cos x|$$

Put $\cos x = t, -\sin x dx = dt$

$$I = -12 \int_1^{-1} |2t^2 + t| dt$$

$$I =$$

$$12 \left(\int_{-1}^{-1/2} (2t^2 + t) dt + \int_{-1/2}^0 -(2t^2 + t) dt + \int_0^1 (2t^2 + t) dt \right)$$

$$I = 17$$



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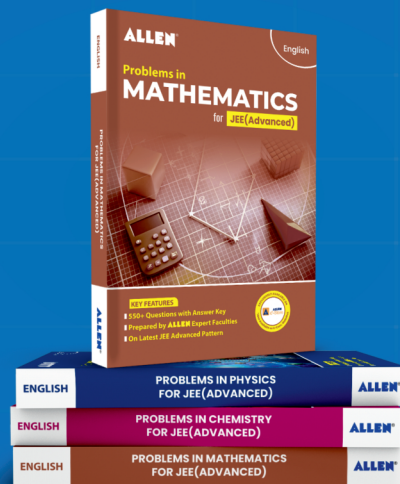
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